Interaction of solitons with localized nonlinear defects

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The interaction of solitons with nonlinear point defects is investigated. Analytical solutions are obtained for bright and dark static solitons placed on the defect. The interaction of propagating solitons with nonlinear defects is studied numerically. Depending on the parameters, the solitons can be trapped, transmitted, reflected or split. A comparison with the case of linear defects is made.

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1. Introduction

A fundamental problem of soliton theory is the interaction with defects and impurities. Widely investigated is the scattering of solitons from point defects [1-4]. Soliton trapping on defects in discrete systems has been studied in [5-7]. Resonant interaction of solitons with extended defects has been obtained in [8,9]. Nonlinear impurity modes have been investigated in [10-12]. In [11], the interaction of optical solitons with strong localized inhomogeneities in the nonlinear and dispersion coefficients has been studied by applying the inverse scattering transform to the output pulse. In the present work we investigate the interaction of solitons with nonlinear point defects by numerical evaluation of the corresponding nonlinear Schrödinger equations.

2. Static solutions

It is instructive to take into account both linear and nonlinear point defects and focus on the differences in the scattering patterns. We shall consider a one-dimensional dynamical system governed by the following perturbed nonlinear Schrödinger equation:

$$i\frac{\partial\alpha}{\partial t}+b\frac{\partial^2\alpha}{\partial x^2}+2g[1+d\delta(x)]|\alpha|^2\,\alpha+\varepsilon\delta(x)\alpha=0, \quad (1)$$

where α is the complex amplitude of the nonlinear wave, and b and g are the dispersion and nonlinear coefficients of the medium. The linear and nonlinear point defects are described by the terms $\varepsilon\delta(x)$ and $d\delta(x)$, respectively. This model describes continuous systems with defects whose size is small compared to the soliton extent, as well as discrete systems with microscopic defects treated in the continuum approximation.

For $d=\varepsilon=0$ and $b/g > 0$ Eq. (1) possesses a fundamental bright soliton solution

$$\alpha(x,t)=\varphi_0\,\mathrm{sech}\frac{x-vt}{L}e^{i(kx-\omega t)}$$
$$\varphi_0^2=\frac{b}{gL^2},\quad k=v/2b,\quad \omega=\frac{v^2}{4b}-\frac{b}{L^2} \quad (2)$$

and for $b/g < 0$ a dark soliton solution

$$\alpha(x,t)=\varphi_0\tanh\frac{x-vt}{L}e^{i(kx-\omega t)}$$
$$\varphi_0^2=-\frac{b}{gL^2},\quad k=v/2b,\quad \omega=\frac{v^2}{4b}+\frac{2b}{L^2}. \quad (3)$$

The parameters L and v are the width and the velocity of the soliton.

We shall consider the influence of impurities on the soliton solutions. The parameters ε and d describe the strength of the defects, which in the present investigation are not considered small. Positive values of ε and d correspond to attraction, while negative values to repulsion.

For $d\neq 0$ and/or $\varepsilon\neq 0$ in the static case ($v = 0$) we have obtained the following solutions of Eq. (1):

$$\alpha(x,t)=\varphi_0\,\mathrm{sech}\left(\frac{|x|}{L}+\Delta\right)e^{-i\omega t} \quad (4)$$

for bright solitons (b/g positive) and

$$\alpha(x,t)=\varphi_0\tanh\left(\frac{|x|}{L}+\Delta\right)e^{-i\omega t} \quad (5)$$

for dark solitons (b/g negative). φ_0 and ω are the same as in (2) and (3), while Δ depends on the type of the defect. For a linear defect ($d=0,\ \varepsilon\neq 0$):

$$\tanh\Delta=\varepsilon L/2b \ \text{ for } b/g>0$$
$$\sinh(2\Delta)=-4b/\varepsilon L \ \text{ for } b/g<0 \quad (6)$$

For a nonlinear defect ($d \neq 0,\ \varepsilon = 0$):

$$\begin{aligned} &\sinh(2\Delta) = 2d/L \ \text{ for } b/g > 0 \\ &\sinh^2 \Delta \tanh \Delta = L/d \ \text{ for } b/g < 0 \end{aligned} \qquad (7)$$

For a defect in both the linear and nonlinear terms ($d \neq 0,\ \varepsilon \neq 0$):

$$\begin{aligned} &L \tanh \Delta - d\left(1 - \tanh^2 \Delta\right) = \varepsilon L^2/2b \ \text{ for } b/g > 0 \\ &d \sinh^2 \Delta \tanh \Delta - \varepsilon L^2 \sinh(2\Delta)/4b = L \ \text{ for } b/g < 0 \end{aligned} \qquad (8)$$

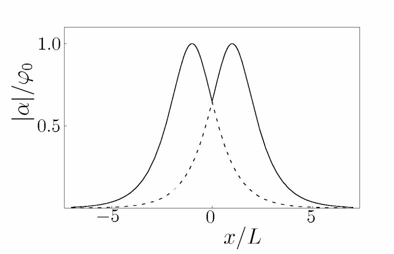

Fig. 1. Solution (4) for Δ = 1 (dashed line) and Δ = -1 (solid line).

If $\Delta > 0$ the function $|\alpha(x)|$ has a single maximum (4) or minimum (5) at $x = 0$. If $\Delta < 0$ the function $|\alpha(x)|$ has two maxima (4) or minima (5) at $x = \pm\Delta L$, respectively (Fig. 1). Δ is much more sensitive to the defect strength in the case of a linear defect and for $\varepsilon L/2b < -1$ the solution (4) decomposes into two non-interacting static solitons.

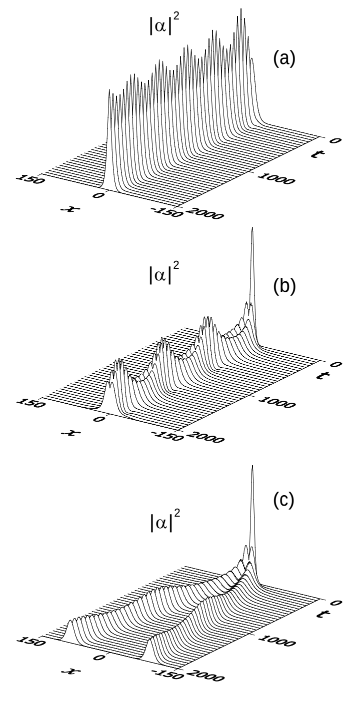

Fig. 2. Evolution of a soliton solution of the form (2) with v=0 placed on a nonlinear impurity with: (a) d = 2; (b) d = -5; (c) d = -7.

Fig. 2 illustrates the evolution of a static pulse of the form (2) which is not an exact solution, placed on a nonlinear impurity. For small defect strengths and for both attractive and repulsive impurities the soliton remains trapped. Slowly-decaying amplitude oscillations are observed, accompanied by a small amount of emitted radiation. The period of the oscillations $2\pi L^2/b$ can be determined from the conservation of the norm of the function which yields

$$L = L_0 (1 - \tanh \Delta)\,, \qquad (9)$$

L_0 being the width of the initial soliton. The calculated period of the oscillations is in good agreement with that estimated from Fig. 2(a). Larger values of the repulsive impurity yield stronger soliton oscillations (Fig. 2(b)) and above a certain threshold a splitting of the soliton into two smaller solitons drifting in opposite directions occurs (Fig. 2(c)).

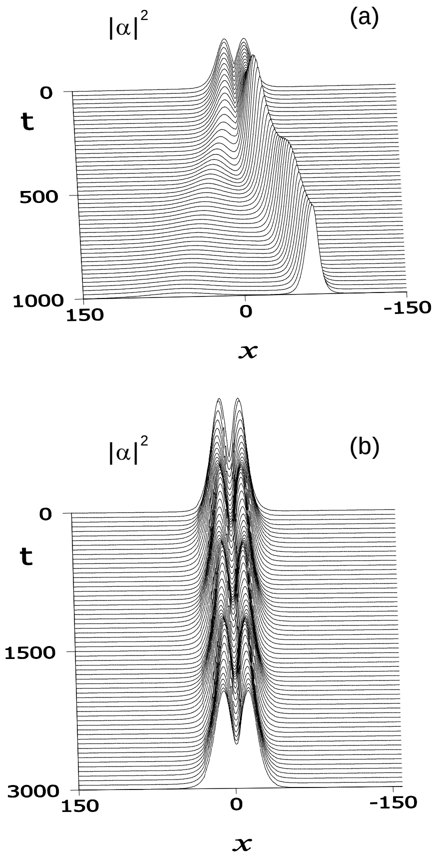

Fig. 3. Evolution of a soliton (4) placed on a repulsive impurity with d N = -15: (a) N = 2(asymmetric perturbation); (b) N = 5(symmetric perturbation).

We investigated also the interaction of the static soliton (4) with a short extended defect with $N < L$. Fig. 3 shows the evolution of the solution (4) for repulsive defects, in which case the interaction with the defects can be interpreted as a perturbation. If the perturbation is asymmetric (the soliton is off the centre of the defect region) the soliton (4) is unstable (Fig. 3(a)), while for symmetric perturbations (the soliton is at the centre of the defect region) the soliton (4) remains stable (Fig. 3(b)). In the case of an attractive defect the soliton is stable for all kinds of perturbation.

3. Scattering of propagating solitons

Of considerable interest is the interaction of propagating solitons with point defects. We investigated numerically the interaction of a soliton of the form (2) with $L = 5.75$ and $v = 0.05$ with a nonlinear point defect. In the case of a repulsive defect, the soliton passes through the defect for $|d| < 0.26$ and is reflected by the defect for $|d| > 0.26$ (Fig. 4).

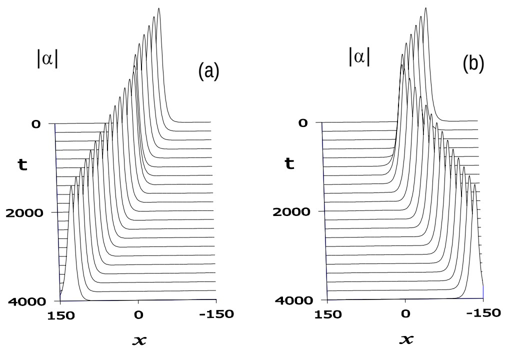

Fig. 4. Scattering of a soliton with v = 0.05 on a repulsive impurity with strength: (a) d = -0.25; (b) d = -0.26.

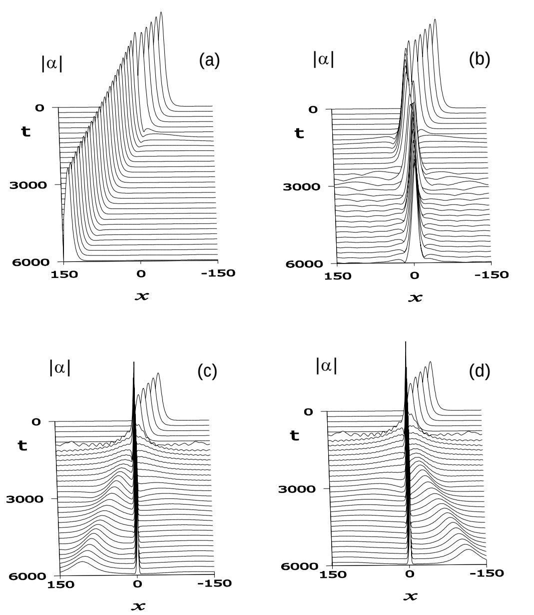

Fig. 5. Interaction of a soliton with v = 0.05 with an attractive impurity with the strength: (a) d = 0.5; (b) d = 0.59; (c) d = 7.0; (d) d = 7.1.

The evolution is more complex in the case of attractive defects. For $d < 0.59$ the soliton passes through the defect with minor perturbations (Fig. 5(a)). For $0.59 \leq d \leq 5.3$ the soliton is trapped by the defect (Fig. 5(b)). The trapped state is accompanied by shape and position oscillations and considerable emission of radiation. For $5.4 \leq d \leq 7.0$ the soliton splits into two solitons: one is transmitted through the defect and the other is trapped (Fig. 5(c)). The reflected soliton is wider and the trapped one is much more narrow. The amplitude, norm and velocity of the transmitted soliton increase with increasing defect strength. However, a small and narrow trapped soliton exists even for very large values of the defect. A noteworthy feature is that the partially-trapped state is not accompanied by continuous radiation, contrary to the fully-trapped state. Another peculiarity is that a partially-trapped state exists even for very large values of the nonlinear defect, contrary to the linear point defect case, where the partially-trapped state disappears for strong enough defects.

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